

## **Reliability and Confidence Interval Analysis of a CMC Turbine Stator Vane**

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High temperature ceramic matrix composites (CMC) are being explored as viable candidate materials for hot section gas turbine components. These advanced composites can potentially lead to reduced weight, enable higher operating temperatures requiring less cooling and thus leading to increased engine efficiencies. However, these materials are brittle and show degradation with time at high operating temperatures due to creep as well as cyclic mechanical and thermal loads. In addition, these materials are heterogeneous in their make-up and various factors affect their properties in a specific design environment. Most of these advanced composites involve two- and three-dimensional fiber architectures and require a complex multi-step high temperature processing. Since there are uncertainties associated with each of these in addition to the variability in the constituent material properties, the observed behavior of composite materials exhibits scatter. Traditional material failure analyses employing a deterministic approach, where failure is assumed to occur when some allowable stress level or equivalent stress is exceeded, are not adequate for brittle material component design. Such phenomenological failure theories are reasonably successful when applied to ductile materials such as metals. Analysis of failure in structural components is governed by the observed scatter in strength, stiffness and loading conditions. In such situations, statistical design approaches must be used. Accounting for these phenomena requires a change in philosophy on the design engineer's part that leads to a reduced focus on the use of safety factors in favor of reliability analyses. The reliability approach demands that the design engineer must tolerate a finite risk of unacceptable performance. This risk of unacceptable performance is identified as a component's probability of failure (or alternatively, component reliability). The primary concern of the engineer is minimizing this risk in an economical manner.

The methods to accurately determine the service life of an engine component with associated variability have become increasingly difficult. This results, in part, from the complex missions which are now routinely considered during the design process. These missions include large variations of multi-axial stresses and temperatures experienced by critical engine parts. There is a need for a convenient design tool that can accommodate various loading conditions induced by engine operating environments, and material data with their associated uncertainties to estimate the minimum predicted life of a structural component.

A probabilistic composite micromechanics technique in combination with woven composite micromechanics, structural analysis and Fast Probability Integration (FPI) techniques has been used to evaluate the maximum stress and its probabilistic distribution in a CMC turbine stator vane. Furthermore, input variables causing scatter are identified and ranked based upon their sensitivity magnitude. Since the measured data for the ceramic matrix composite properties is very limited, obtaining a probabilistic distribution with their corresponding parameters is difficult. In case of limited data, confidence bounds are essential to quantify the uncertainty associated with the distribution. Usually 90 and 95% confidence intervals are computed for material properties. Failure properties are then computed with the confidence bounds. Best estimates and the confidence bounds on the best estimate of the cumulative probability function for R-S (strength – stress) are plotted. The methodologies and the results from these analyses will be discussed in the presentation.

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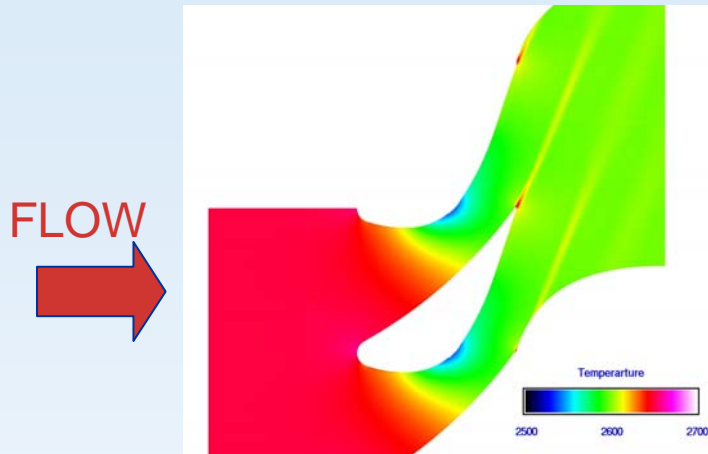
# Presentation Outline

- Background
- Vane stress analysis
- Probabilistic analysis theory and approach
- Vane risk assessment
- Confidence interval analysis
- Results/Discussions
- Summary

# Background

- **UEET Goal: Design/Fabricate/Analyze/Test an all CMC turbine stator vane capable of withstanding 100 hours life in a simulated engine environment in High Pressure Burner Rig at NASA Glenn**
  - Successful demonstration reported earlier
  - Performed deterministic thermal/stress analyses of vane and results reported earlier
- **Benefits: Allows higher combustor exit temperatures**
  - 2700 °F surface temperature capability goal
  - Potential cooling requirement reduction of 15-25%

# CMC Turbine Vane Analysis

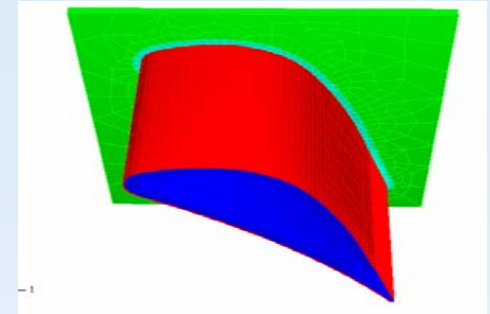


- **CFD Analysis for HPBR Environment**

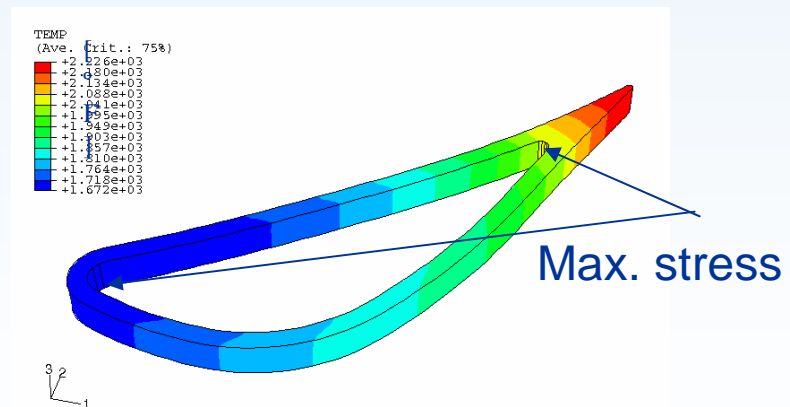


- **6 ply Sylramic Fiber Vane**

- **Goal:**
- Prediction of the temperature and stress conditions present in the mid-span of the vane.
- **Approach:**
- Flow boundary conditions modeled via CFD analysis.
- Attachment concept approximated.
- Vane without internal rib was modeled.

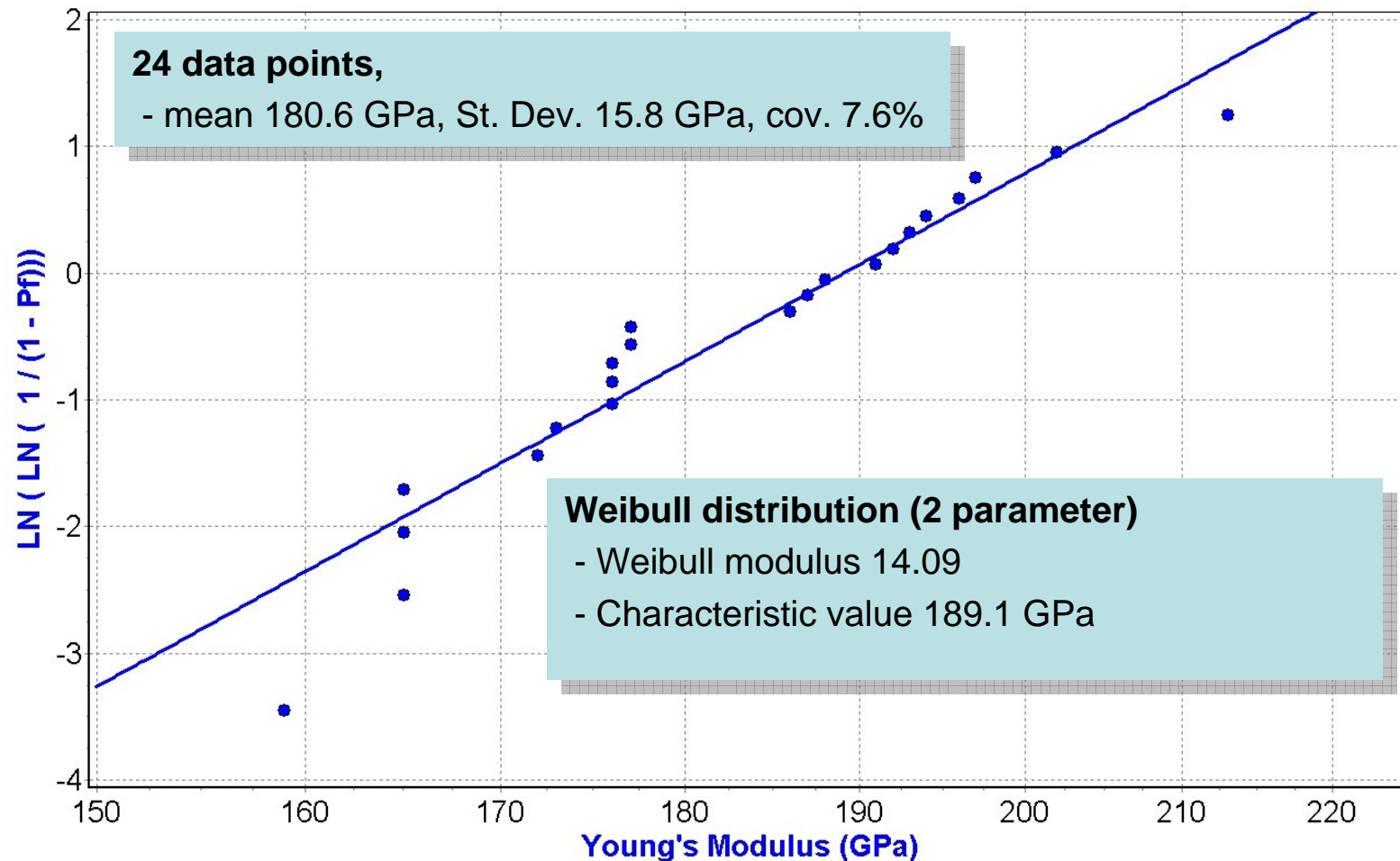


- **Thermal/Stress Analysis of Turbine Vane**

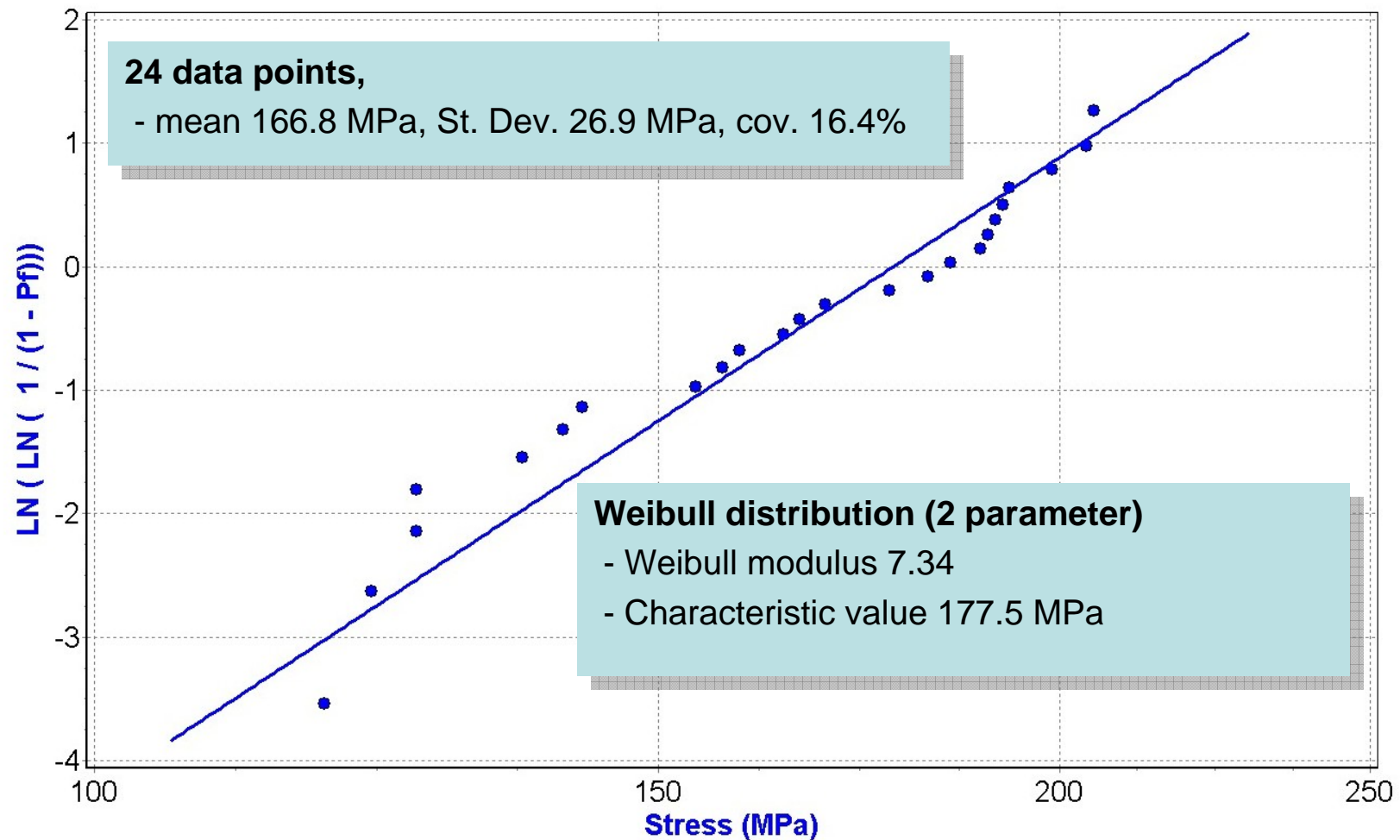


- **Hoop-stress for Mid-span**

# In-plane Modulus of MI SiC/SiC Material at 2200 °F



# Proportional Limit of MI SiC/SiC Material at 2200 °F





# Vane Risk Assessment

- Perform a formal risk assessment (using probabilistic methodology) of the all CMC turbine stator vane under high pressure burner rig conditions.
- Account for the observed scatter in material behavior (modulus and strength).
- Consider possible uncertainties in loading conditions (external and internal pressures), and other material properties (Poisson's ratios and thermal expansion coefficients).
  - *Note: Risk/failure for the present purposes is defined as not meeting the design requirements. Vane is designed to assure that the stresses under rig conditions are always below the proportional limit.*

# Probabilistic Distribution Function Computational Details

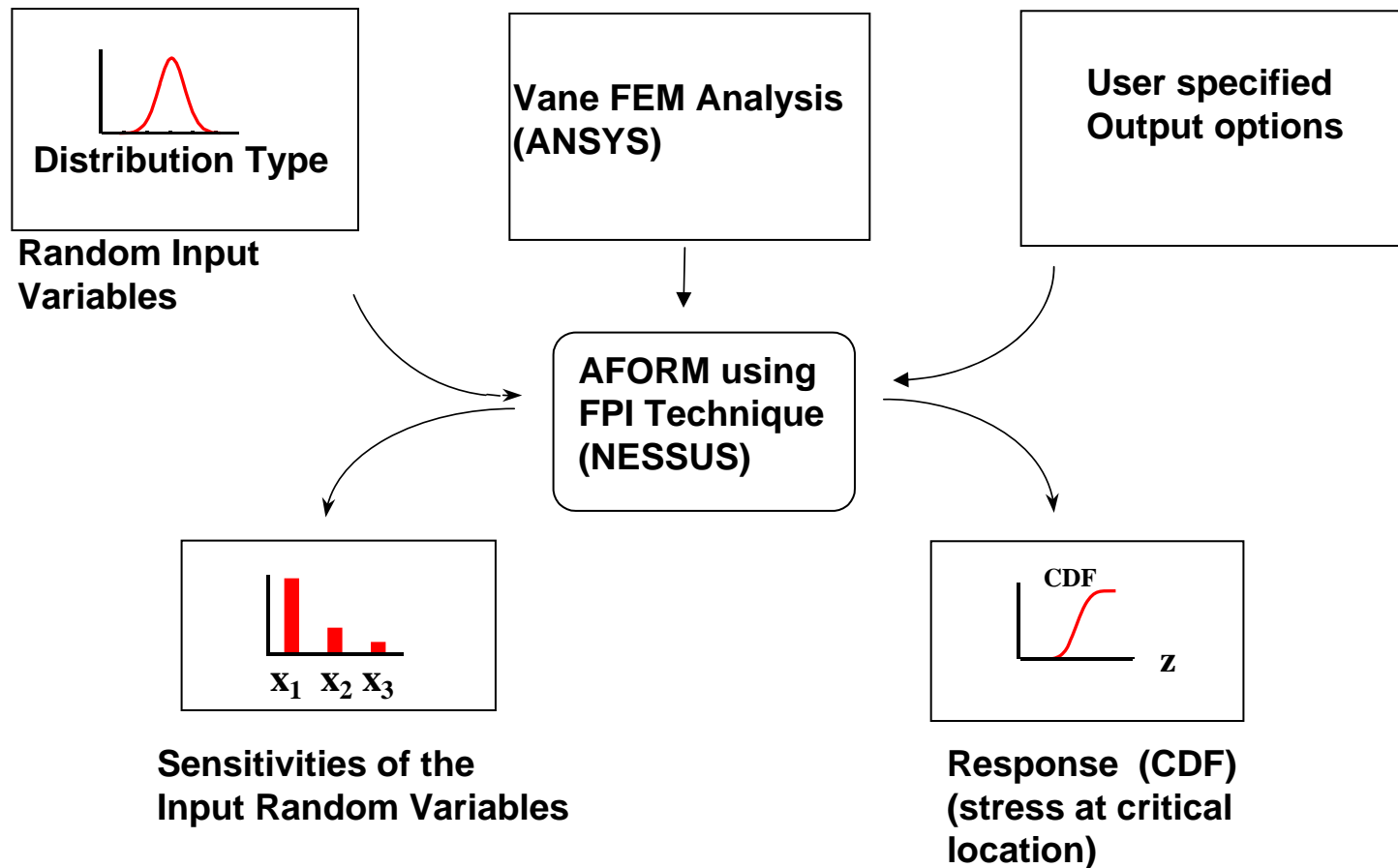
$$S(\underline{X}) = S(X_1, X_2, \dots, X_n)$$

$$g(\underline{X}) = g(R, S) = R - S$$

$$P_f = P[g(\underline{X}) \leq 0] = \int_{\Omega} \dots \int f_x(x) dx$$

Where S is response variable (Stress at a critical location),  
X Input random variable vector (properties, geometry, loads,  
etc.,  $P_f$  probability of failure,  $f_x$  joint probability density  
function

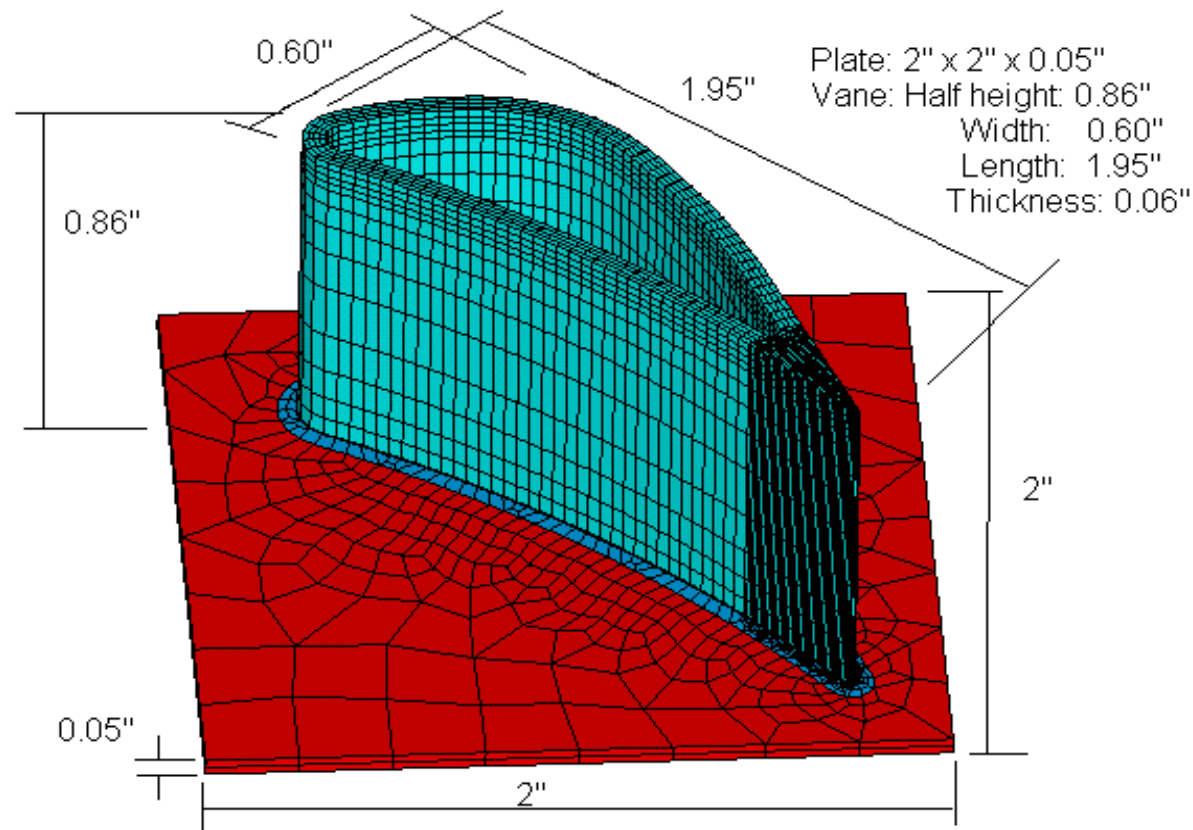
# Reliability Assessment Flow Chart



# Finite Element Model

## Finite Element Model of a Stator Vane Test

Nodes: 15,900; Elements: 12,385 Eight-Node Brick



# Cases Studied

## 1. Material Variability

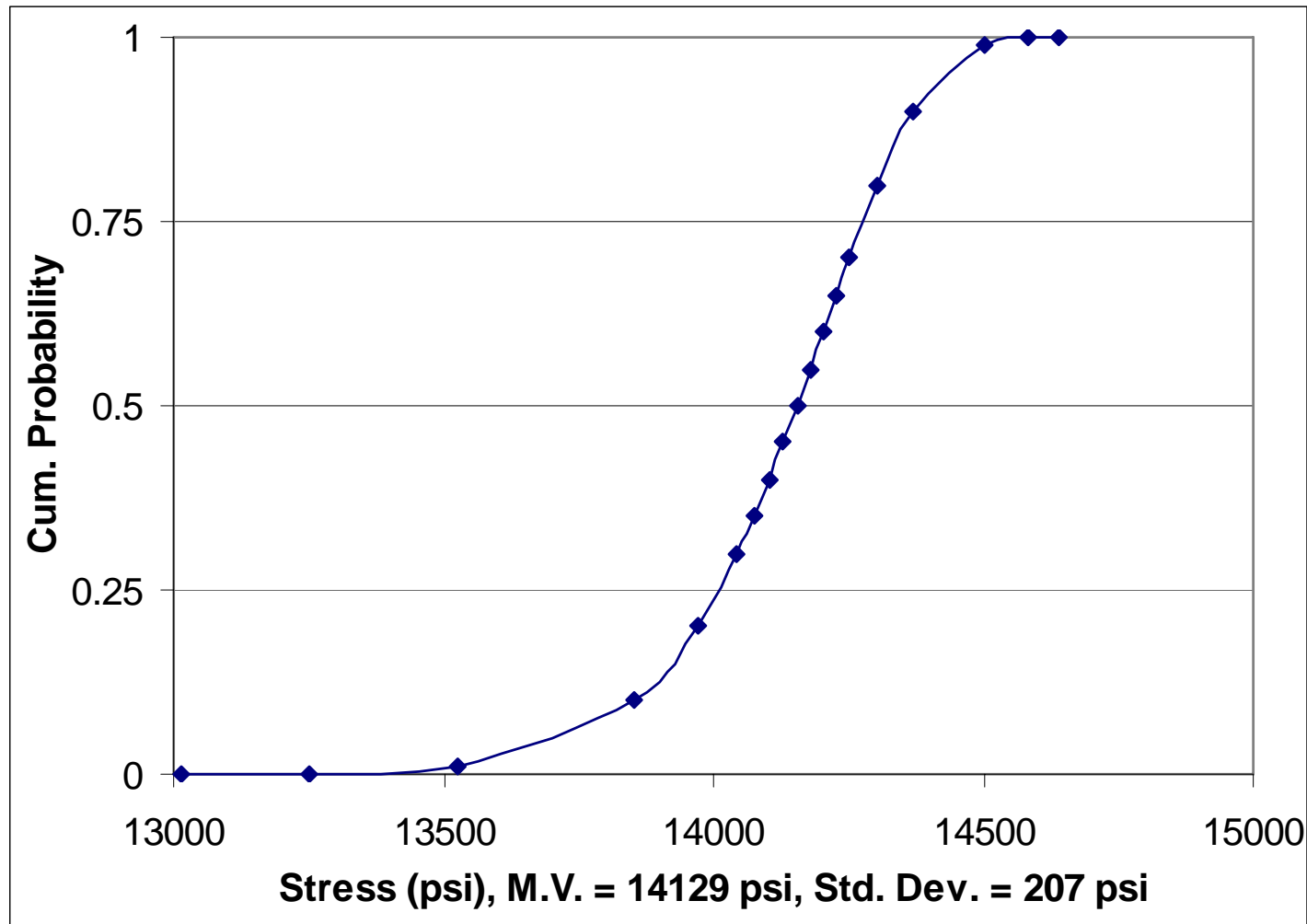
- Modulus statistics are taken based on measured data
- Poisson's ratio: Normal distribution with 2% coefficient of variation
- Thermal expansion coefficients: Normal distribution with 8% variation

## 2. Material Variability plus Loads variability

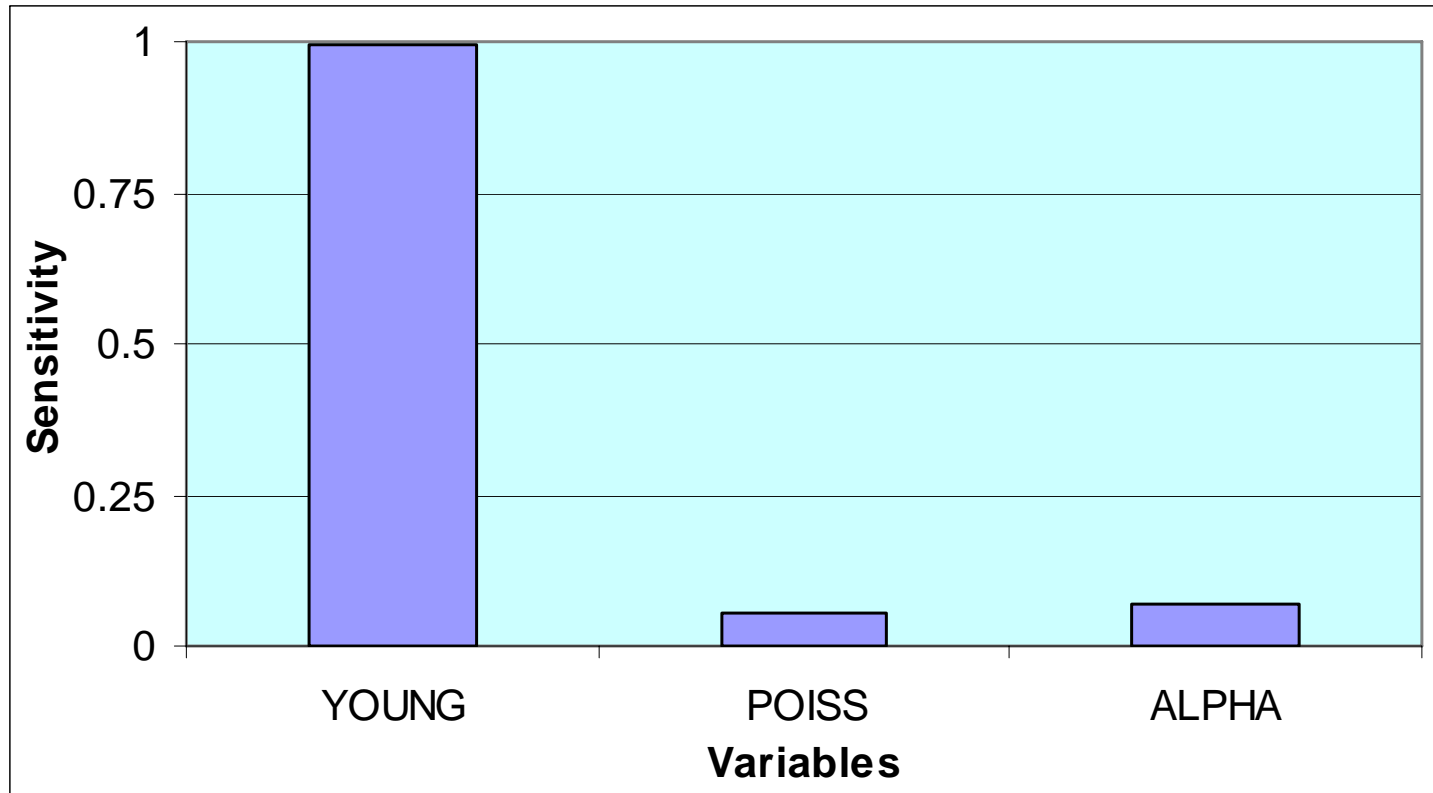
- Internal pressure mean 125 psi, cov. 4%, Normal distribution
- External pressure mean 80 psi, cov. 8%, Normal distribution

*Note: Only For Modulus and Strength we have measured data.  
For the other variables we have used assumed distributions.*

# Case 1: CDF of Hoop Stress at the Critical Location

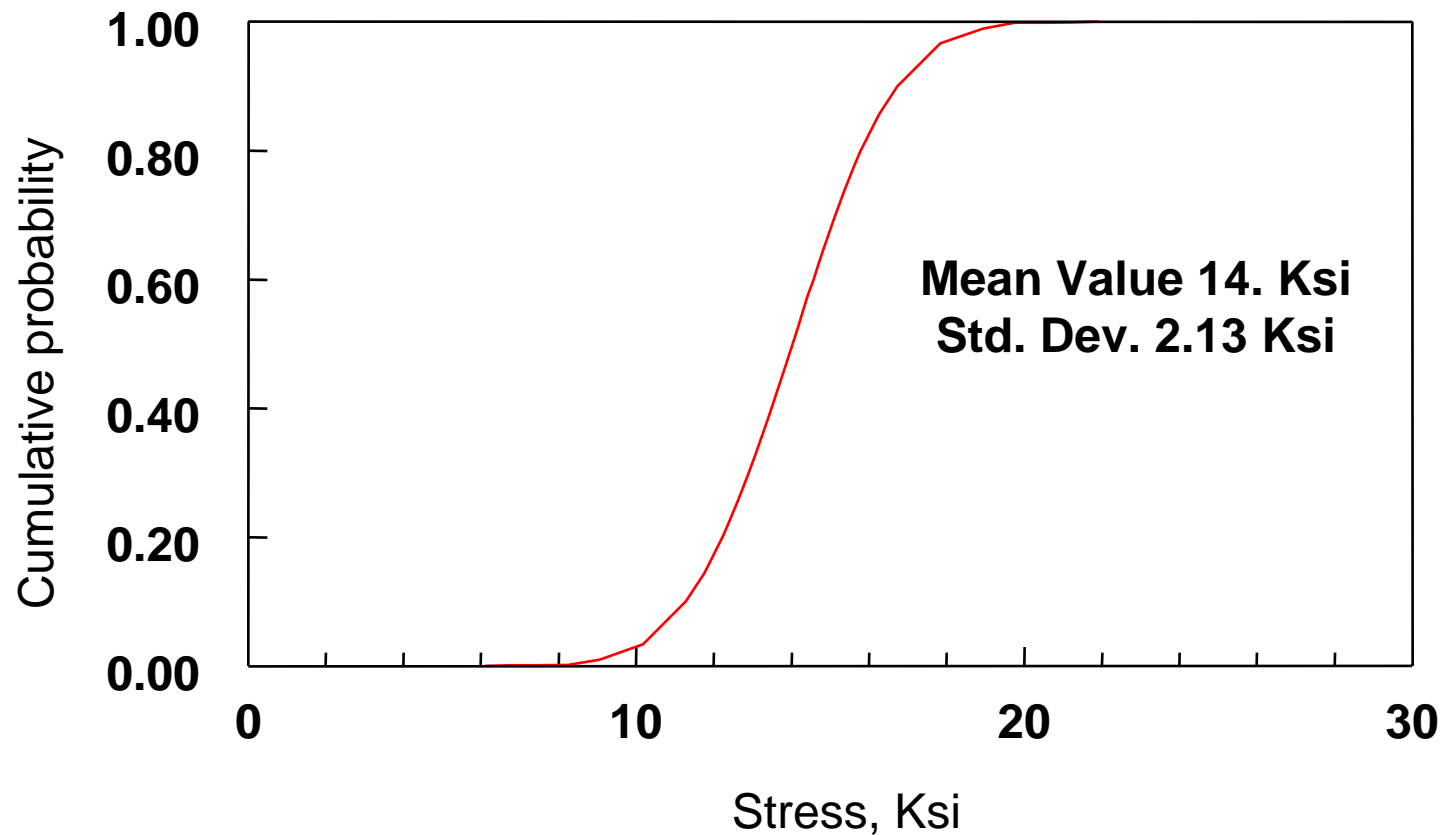


# Sensitivity Factors for Hoop Stress



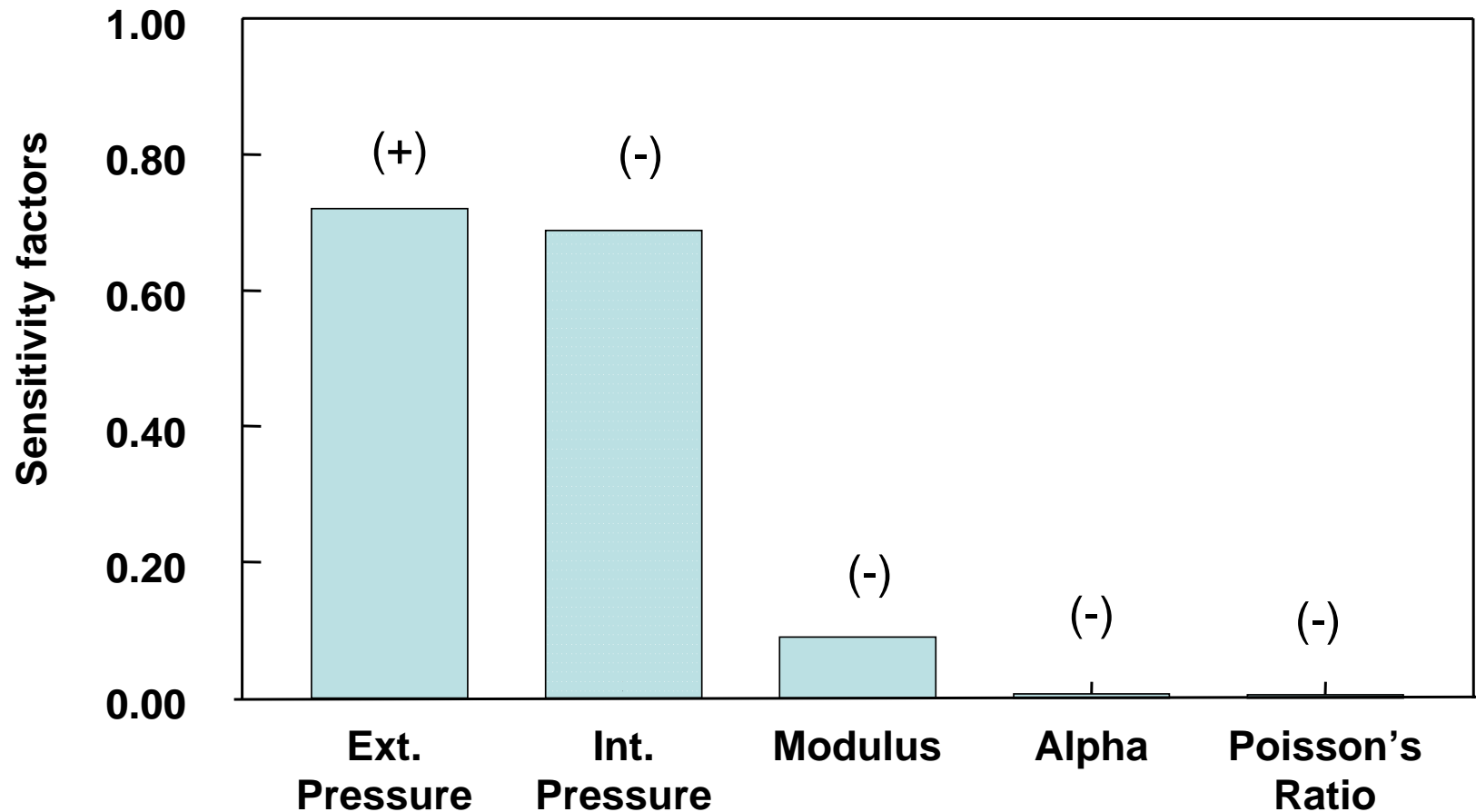
- 1. Stiffness is the most influencing variable contributing to the scatter in stress response.***
- 2. Poisson's ratio, and Coefficient of expansion can be treated as deterministic variables.***

# Case 2: CDF of Hoop Stress at Critical Location





# Ranking of Scatter Causing Input Variables on Stress Response



- 1. Variability in loads affects most the probabilistic (scatter in) stress response.***
- 2. Among the Material variables, stiffness is by far the most significant one. Alpha, and Poisson's Ratio may be neglected***

# Confidence Levels

- Quantities that enter into reliability computations (stress, strengths etc.) are usually based on experimental/field data or derived from other quantities.
- Limited amount of data introduces an uncertainty in probability distribution functions leading to uncertainty in the failure probability (reliability) computations.
- The larger the amount of data, the lower the uncertainty about its distribution and higher the confidence one can place on the computed reliability.
- In case of limited data, confidence bounds are essential to quantify the uncertainty associated with the distribution.
- One way to express uncertainty about a computed reliability is in terms of lower one-sided confidence limits. The reliability is associated with a confidence level. For example, reliability  $R$  at  $K\%$  confidence level means that there is  $K\%$  chance (probability) that the exact value of reliability is not less than  $R$ .

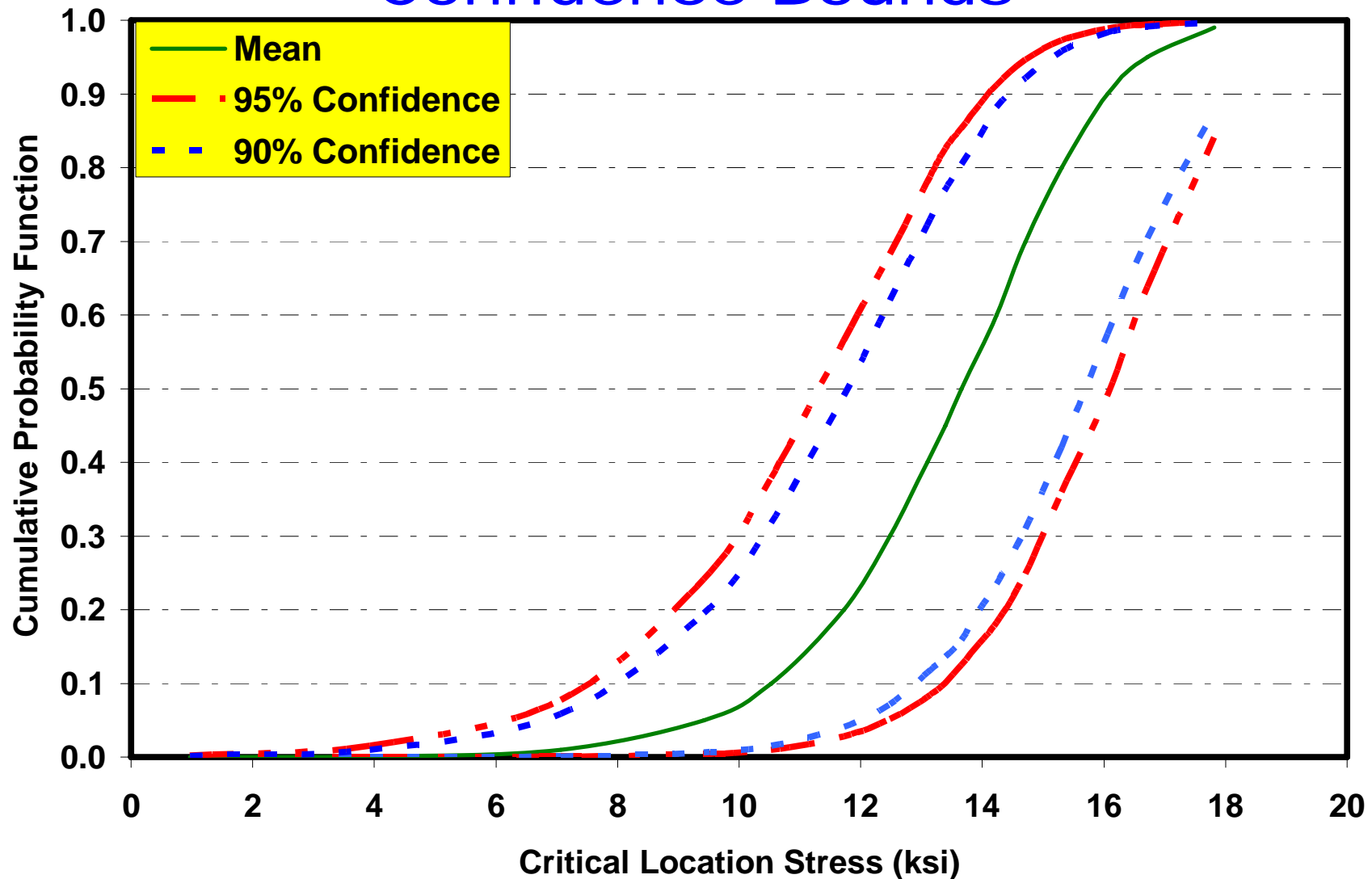
*Note: The remaining charts show how uncertainty in results can be bound with confidence intervals and the reliability can be quoted with upper and lower 90/95% confidence bounds*

# Parameters for Confidence Interval Analysis

Parameter		Best Estimate	95% - Lower	95% - Upper
CMC Modulus (Msi)	Mean value	26.2	25.2	27.3
	Std. deviation	2.3	1.84	2.97
CMC Strength (Ksi)	Mean value	24.2	22.4	25.9
	Std. deviation	3.9	3.05	4.84

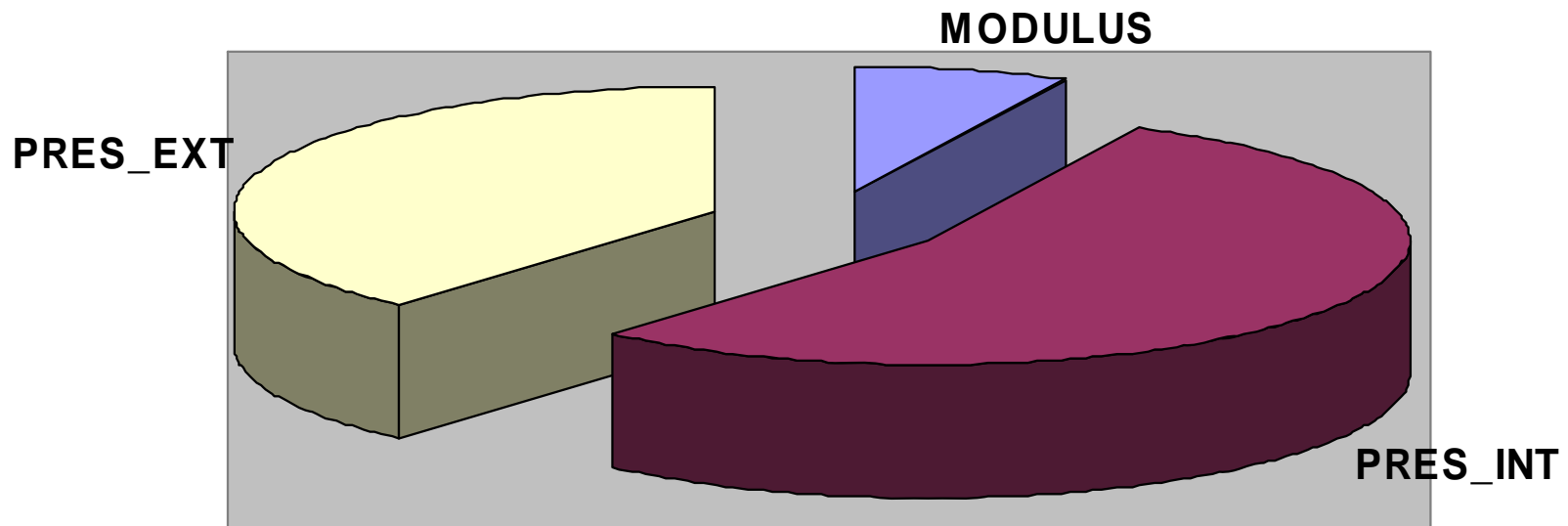
- 1. The confidence bounds for Measured Data are established using MLE Weibull fits in Matlab program. From the estimated Weibull parameters, means and standard deviations as well as the confidence bounds are established.***

# Maximum Hoop Stress with Confidence Bounds



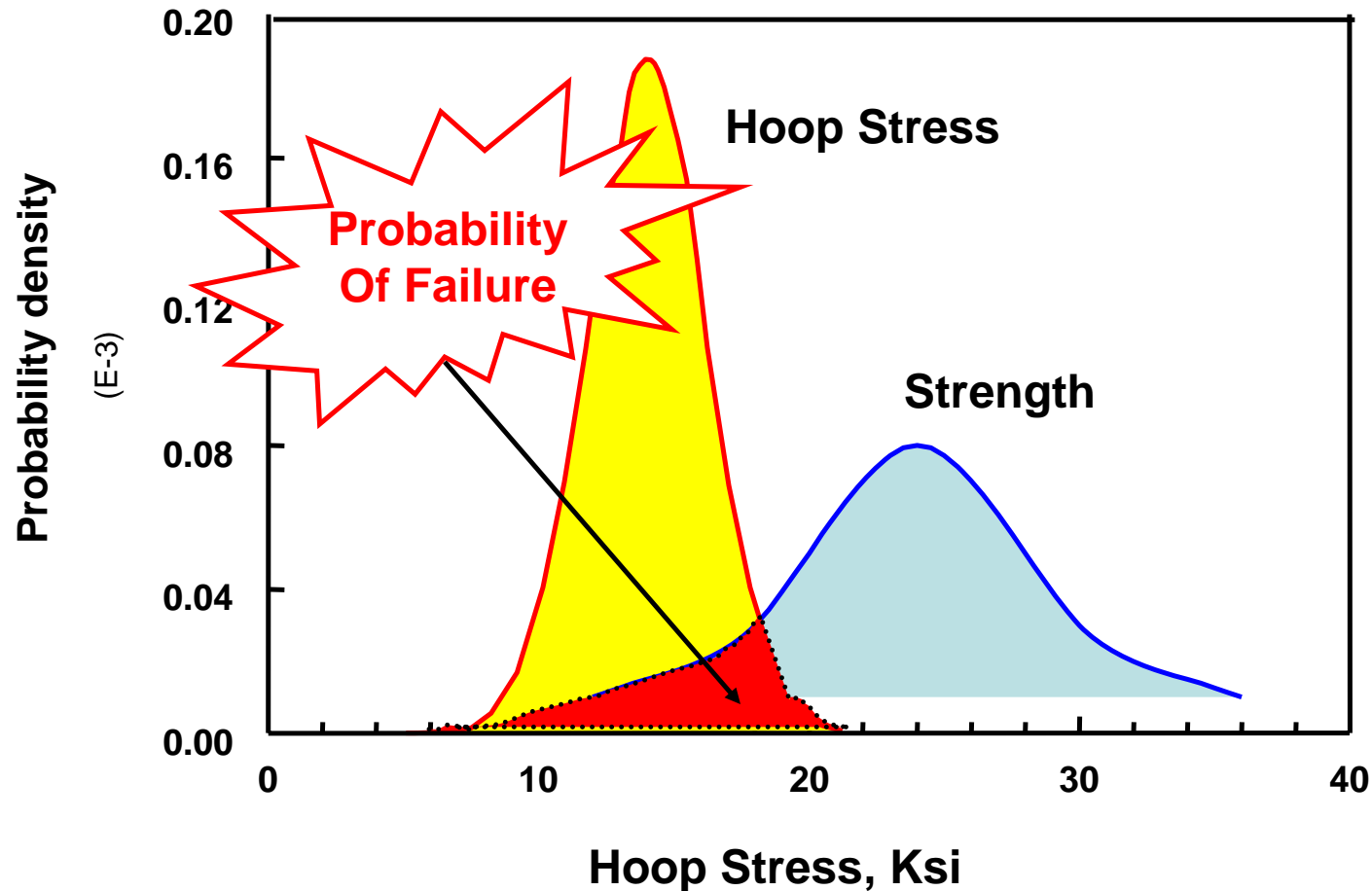
***Note: NESSUS 8.3 (SWRC) is used to obtain the CDFs with Confidence Bounds***

# Sensitivity Factors (Best Estimates at 0.9 Probability)

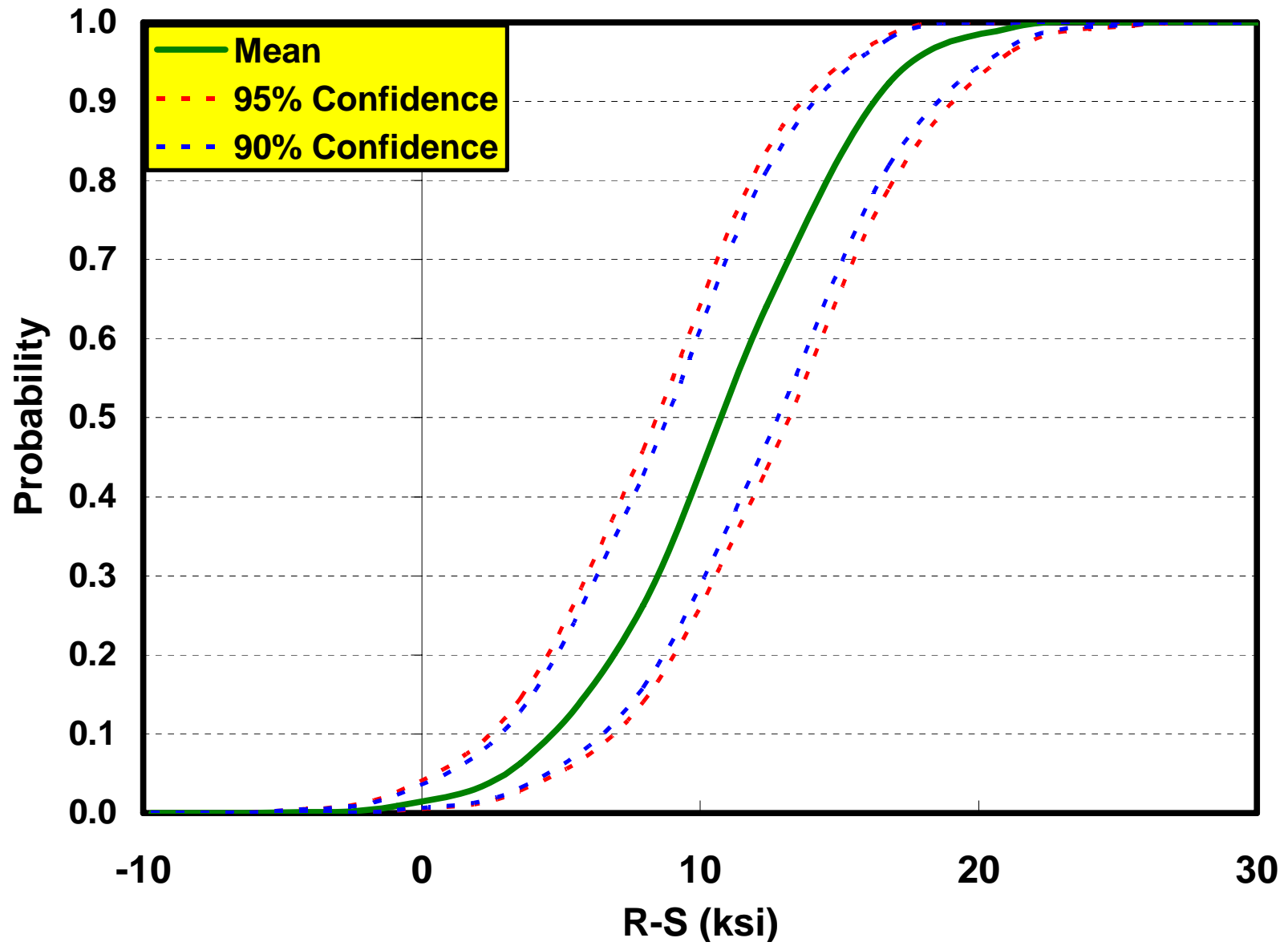


***Variability in loads affects most the probabilistic (scatter in) stress response.***

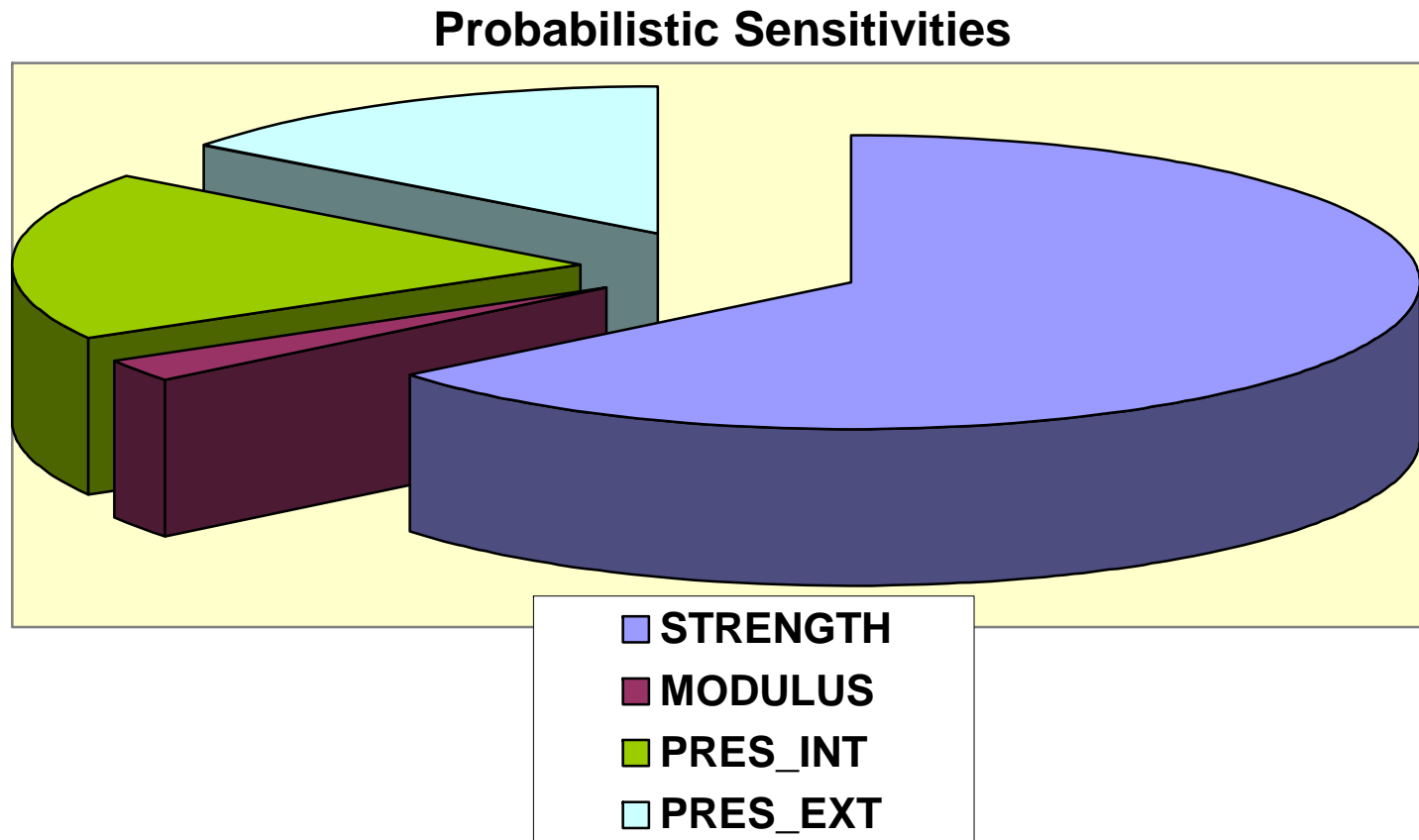
# Risk Assessment Based on Failure to Meet Design Requirements



# Vane Reliability with Confidence Bounds



# Sensitivity Factors at $Z$ (R-S)=0



***Variability in strength affects most the Vane Reliability (failure) followed by the scatter in Loads***



# Failure Analysis Summary

<b>Z=R-S</b>	<b>Best Est.</b>	<b>95% - Lower</b>	<b>95% - Upper</b>	<b>90% - Lower</b>	<b>90% - Upper</b>
<b>0</b>	<b>0.014</b>	<b>0.005</b>	<b>0.039</b>	<b>0.006</b>	<b>0.034</b>
<b>Reliability</b>	<b>0.986</b>	<b>0.995</b>	<b>0.961</b>	<b>0.994</b>	<b>0.966</b>
<b>Reliability in # of “nines” [# = <math>-\log_{10}(1-P_f)</math>]</b>	<b>1.9</b>	<b>2.3</b>	<b>1.4</b>	<b>2.2</b>	<b>1.5</b>

# Summary

- **With the aid of ANSYS/FEM and In house computational tools (NESSUS) a formal reliability assessment of an all CMC turbine stator vane is completed**
  - Material variability only, and Material + Loads variability considered with Confidence bounds on the input variables
  - Stress response and Failure Response evaluated at various confidence bounds (90 and 95%)
- **Sensitivity of random variables indicated that**
  - loads have the most dominating effect on the critical location stress response
- **Scatter in strength is about the most important variable in dictating the reliability of the design.**
  - Improvements in fabrication can lead to reduction in the observed scatter in proportional limit thereby improving the reliability
  - Variability in critical stresses can be effectively controlled by having tighter controls in pressures.
- **Confidence interval analysis is essential to bound the uncertainty in reliability due to material/loads variability and limited amount of available data**